RED BUILDING TEAM

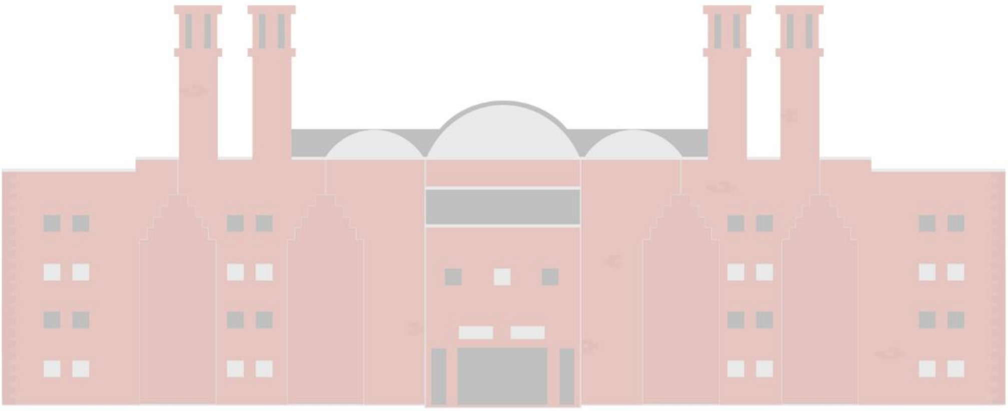
قسم هندسة الحواسيب والأتمتة

السنة الثانية / الفصل الأول

التحليل الرياضي ٣

المحاضرة الحادي والعشرون

٧ صفحات



الدكتور: معاذ عبد المجيد

التاريخ: ١٦/١١/٢٠١٤

$$I_2=\int_0^{\frac{\pi}{2}}\cos^{2a-1}\varphi\sin^{2b-1}\varphi\,d\varphi=\frac{1}{2}\beta(a,b)\ \ldots\boxed{*}$$

$$\sin^2\varphi=Y$$

$$2\sin\varphi\cos\varphi\,d\varphi=dY$$

$$\sin^{2b-2}\varphi=Y^{b-1}\quad,\quad\cos^{2a-2}\varphi=(\cos^2\varphi)^{a-1}=(1-Y)^{a-1}$$

$$\mathrm{I}_2=\frac{1}{2}\int_0^1(1-Y)^{a-1}Y^{b-1}dY=\frac{1}{2}\beta(a,b)$$

صيغ واليس : $\int_0^{\frac{\pi}{2}}\cos^{2a-1}\rho\ \sin^{2b-1}\rho\ d\rho=\frac{1}{2}\beta(a,b)$

$$2b-1=0\quad,\quad 2a-1=\ \Rightarrow a=\frac{p+1}{2}\quad,b=\frac{1}{2}$$

$$\int_0^{\frac{\pi}{2}}\cos^p\rho\,d\rho=\frac{1}{2}\beta\left(\frac{p+1}{2},\frac{1}{2}\right)=\frac{1}{2}\frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{p}{2}+1\right)}=\int_0^{\frac{\pi}{2}}\sin^p\rho\,d\rho$$

$$\Gamma(\mathrm{a})\Gamma(\mathrm{b})=2\Gamma(\mathrm{a}+\mathrm{b})\int_0^{\frac{\pi}{2}}\cos^{2a-1}\rho\ \sin^{2b-1}\rho\ d\rho$$

$$\frac{\Gamma(\mathrm{a})\Gamma(\mathrm{b})}{2\Gamma(\mathrm{a}+\mathrm{b})}=\int_0^{\frac{\pi}{2}}\cos^{2a-1}\rho\ \sin^{2b-1}\rho\ d\rho$$

$$2b-1=p\quad,\quad 2a-1=0\quad\Rightarrow a=\frac{1}{2}\quad,b=\frac{p+1}{2}$$

$$\int_0^{\frac{\pi}{2}}\cos^p\rho\,d\rho=\int_0^{\frac{\pi}{2}}\sin^p\rho\,d\rho=\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{p+1}{2}\right)}{2\Gamma(\frac{\mathrm{p}}{2}+1)}=\frac{\sqrt{\pi}}{2}\frac{\Gamma\left(\frac{p+1}{2}\right)}{\Gamma(\frac{\mathrm{p}}{2}+1)}$$

P=2nعدد زوجي

$$\int_0^{\frac{\pi}{2}}\cos^{2n}\rho\,d\rho=\int_0^{\frac{\pi}{2}}\sin^{2n}\rho\,d\rho=\frac{\sqrt{\pi}}{2}\frac{\Gamma\left(n+\frac{1}{2}\right)}{\Gamma(\mathrm{n}+1)}=\frac{\sqrt{\boldsymbol{\pi}}}{\mathbf{2}}\frac{(\mathbf{2}\boldsymbol{n}-\mathbf{1})!!}{\mathbf{2}\boldsymbol{n}!!}$$

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P=2n-1عدد فردي

$$\int_0^{\frac{\pi}{2}} \cos^{2n-1}\rho\, d\rho = \int_0^{\frac{\pi}{2}} \sin^{2n-1}\rho\, d\rho = \frac{\sqrt{\pi}}{2}\,\frac{\Gamma(n)}{\Gamma(n+\frac{1}{2})} = \frac{\sqrt{\pi}}{2}\,\frac{(n-1)!}{\frac{\sqrt{\pi}}{2^n}\,(2n-1)!!}$$

$$= \frac{\mathbf{2^{n-1}(n-1)!}}{\mathbf{(2n-1)!!}}$$

مثال: أحسب التكامل التالي:

$$\int_0^{\frac{\pi}{2}} \cos^{27}\rho\, d\rho$$

$$p = 27 \Rightarrow 2n-1 = 27 \Rightarrow n = 14$$

$$I = \frac{26!!}{27!!}$$

مثال: احسب التكامل التالي

$$A = \int_0^5 x^4\sqrt{(25-x^2)}\; dx$$

الحل: لنفرض ان

$$\left(\frac{x}{5}\right)^2 = y \Rightarrow 5\sqrt{1-\left(\frac{x}{5}\right)^2} = 5\sqrt{1-y} = 5(1-y)^{\frac{1}{2}}$$

$$x = 5\sqrt{y} \;\Rightarrow dx = \frac{5}{2}\frac{dy}{\sqrt{y}}$$

$$A = \int_0^1 5^4\, y^2\, 5(1-y)^{\frac{1}{2}}\,\frac{5}{2}\frac{dy}{\sqrt{y}} = \int_0^1 \frac{5^6}{2}\, y^{\frac{3}{2}}\,(1-y)^{\frac{1}{2}}\, dy = \frac{5^6}{2}\beta\left(\frac{5}{2},\frac{3}{2}\right)$$

$$a-1 = \frac{3}{2} \Rightarrow a = \frac{5}{2}$$

$$b-1=\frac{1}{2}\Rightarrow b=\frac{3}{2}$$

$$A=\frac{5^6}{2}\,\frac{\Gamma\left(\frac{5}{2}\right)\Gamma\left(\frac{3}{2}\right)}{\Gamma(4)}=\frac{5^6}{2}\left(\frac{\frac{\sqrt{\pi}}{2}\left(\frac{3\sqrt{\pi}}{4}\right)}{3!}\right)\Rightarrow A=\frac{5^6}{2^5}\,\pi$$

مثال: احسب التكامل

$$\boldsymbol{I=\int_0^{\frac{\pi}{2}}\cos^m x\ \sin^n x\ dx}$$

الحل:

$$t=\sin^2 x\ \Rightarrow \sin^n x=t^{\frac{n}{2}}$$

$$\cos^2 x=1-\sin^2 x=1-t$$

$$dt=2\sin x\cos x\,dx$$

$$\cos^{m-1}x=(1-t)^{\frac{m-1}{2}}$$

$$I=\int_0^1\frac{1}{2}\,t^{\frac{n-1}{2}}\,(1-t)^{\frac{m-1}{2}}\,dt=\frac{1}{2}\,\beta\left(\frac{n+1}{2},\frac{m+1}{2}\right)$$

مثال : احسب التكامل

$$\boldsymbol{I=\int_0^{\frac{\pi}{2}}\cos^4 x\ \sin^7 x\ dx}$$

$$=\frac{1}{2}\,\beta\left(\frac{5}{2},4\right)=\frac{\Gamma\left(\frac{5}{2}\right)\Gamma(4)}{2\Gamma\left(\frac{13}{2}\right)}$$

rbd
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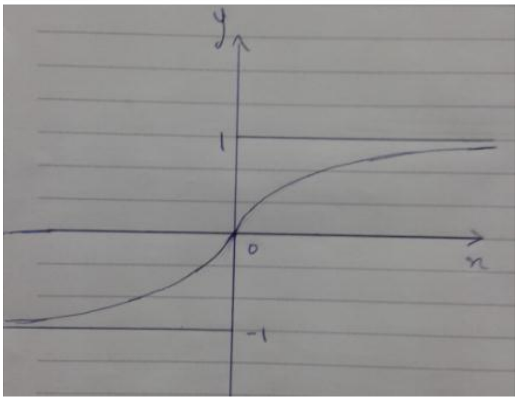
٣. تابع الخطأ: $error\ f$

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

خواصه:



١- تابع فردي $erf(-x) = -erf(x)$

٢- تابع متزايد $= \left(\frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt\right)' = [erf(x)]'$

$\frac{2}{\sqrt{\pi}} e^{-x^2} > 0 \,;\, \forall x$

٣- $erf(0) = 0$

٤- $erf(\infty) = \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-t^2}\, dt = \frac{2}{\sqrt{\pi}} \frac{\sqrt{\pi}}{2} = 1$

٥- $erf(-\infty) = -1$

التابع المتمم لتابع الخطأ:

$$erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2}\, dt$$

$$erf(x) + erfc(x) = \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-t^2}\, dt = 1$$

٤. تكاملي فرينيل:

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2} t^2\right) dt$$

$$C(x) = \int_0^x \cos\left(\frac{\pi}{2} t^2\right) dt$$

هما تابعين فرديين في x

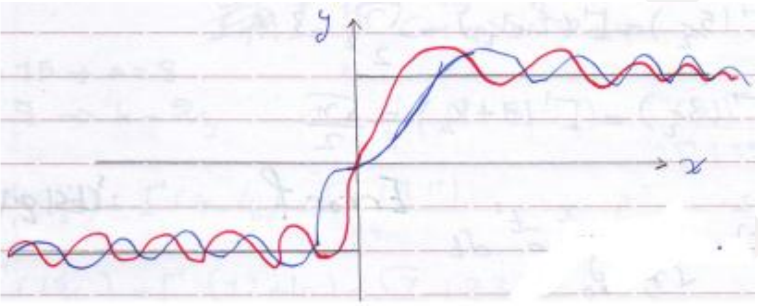
$$S(-x) = -S(x)$$

$$C(-x) = -C(x)$$

$$S(0) = C(0) = 0$$

$$S(\infty) = C(\infty) = \int_{0}^{\infty} \sin(\frac{\pi}{2} t^2)dt = \frac{1}{2}$$

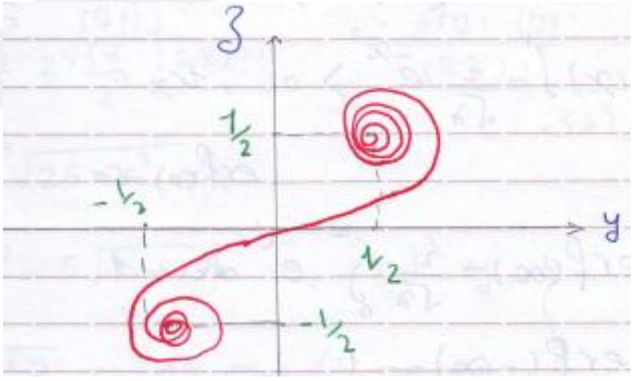
$$S(-\infty) = C(-\infty) = -\frac{1}{2}$$



باعتبار x وسيط في التكاملين

يسمى حلزون كورنو

$$y = C(x) \ , z = S(x)$$



٥. الجيب والتجيب التكاملي:

تابع الجيب التكاملي $Si(x) = \int_0^x \frac{\sin t}{t} dt$

$$Si(-x) = -Si(x)$$

$$Si(0) = 0 \quad , \quad Si(\infty) = \int_0^\infty \frac{\sin t}{t}\, dt = \frac{\pi}{2}$$

المتمم له:

$$si(x) = \int_x^\infty \frac{\sin t}{t} dt$$

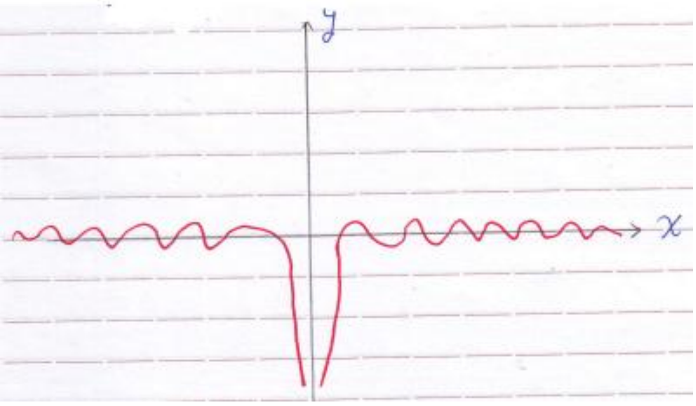
$$Si(x) + si(x) = \frac{\pi}{2}$$

تابع التجيب التكاملي $\quad Ci(x) = -\int_x^\infty \frac{\cos t}{t} dt$

$$Ci(0) = -\int_0^\infty \frac{\cos t}{t}\, dt = -\infty$$

$$Ci(\infty) = -\int_\infty^\infty \frac{\cos t}{t}\, dt = 0$$

ليس له متمم



مثال:

$$A=\int_{0}^{\infty} x^2\, e^{-2x^2} dx$$

الحل:

$$2x^2=t \Rightarrow x^2=\frac{1}{2}t \Rightarrow 2xdx=\frac{1}{2}dt$$

$$x=\frac{1}{\sqrt{2}}t^{\frac{1}{2}} \quad , \quad dx=\frac{dt}{4x} \quad =\frac{dt}{\frac{4}{\sqrt{2}}t^{\frac{-1}{2}}}$$

$$A=\int_{0}^{\infty}\frac{1}{2}t\,e^{-t}\frac{dt}{\frac{4}{\sqrt{2}}t^{\frac{1}{2}}}=\frac{\sqrt{2}}{8}\int_{0}^{\infty}t^{\frac{1}{2}}\,e^{-t}dt$$

$$A=\frac{\sqrt{2}}{8}\Gamma\left(\frac{3}{2}\right)=\frac{\sqrt{2}}{8}\cdot\frac{\sqrt{\pi}}{2}$$

مثال:

$$A=\int_{0}^{\infty} 3^{-4x^2} dt$$

الحل:

$$3^{-4x^2}=e^{-t} \Rightarrow -4x^2\ln 3=-t$$

$$A=\int_{0}^{\infty} e^{-t}\frac{t^{-\frac{1}{2}}\,dt}{\sqrt{ln3}\,4}$$

$$\frac{1}{4\sqrt{\ln 3}}\int_{0}^{\infty} e^{-t}\,t^{-\frac{1}{2}}dt=\frac{1}{4\sqrt{\ln 3}}\Gamma\left(\frac{1}{2}\right)=\frac{1}{4}\sqrt{\frac{\pi}{\ln 3}}$$

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